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A STUDY ON THE INVERSE GALOIS PROBLEM IN GALOIS THEORY & APPLIED MATHEMATICS

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INTRODUCTION

In mathematics, more specifically in abstract algebra, **Galois theory**, named after Évariste Galois, provides a connection between field theory and group theory. Using Galois theory, certain problems in field theory can be reduced to group theory, which is in some sense simpler and better understood.

Originally Galois used permutation groups to describe how the various roots of a given polynomial equation are related to each other. The modern approach to Galois theory, developed by Richard Dedekind, Leopold Kronecker and Emil Artin, among others, involves studying automorphisms of field extensions.

Further abstraction of Galois theory is achieved by the theory of Galois connections.

Galois Theory is the algebraic study of groups that can be associated with polynomial equations. This book covers the basic material of Galois theory and discusses many related topics, such as Abelian equations, solvable equations of prime degree, and the casus irreducibilis, that are not mentioned in most standard treatments. It also describes the rich history of Galois theory, including the work of Lagrange, Gauss, Abel, Galois, Jordan, and Kronecker.

Cox (mathematics, Amherst College) covers both classic applications of the theory and some of the more novel approaches. He begins with polynomials in the theory's foundations, cubic equations, advancing to symmetric polynomials and the roots of polynomials. He proceeds explaining fields, including extension fields, normal and separable extensions, the Galois group, and Galois correspondence. Topics in applications include solvability by radicals, cyclotomic extensions, geometric constructions and finite fields. He then considers the work of Lagrange, Galois and Kronecker in concert, the process of computing Galois groups, solvable permutation groups, and the lemniscate, including the lemniscatic function, complex multiplication and Abel's theorem.

PURE MATHEMATICS AND APPLIED MATHEMATICS

There are two separate disciplines within the general subject area – Pure Mathematics and Applied Mathematics. **You may choose units from either or both disciplines.**

If you enjoy problem solving, working with computers and using your mathematics to deal with real applications in science, engineering, economics and biology then you should consider enrolling in some **Applied Mathematics** units. You will attain a high level of mathematical expertise and a good deal of practical computer experience, both of which will stand you in good stead in a wide variety of possible careers, for example in computing, finance, telecommunications, and mathematics research.

If you appreciate the elegance of conceptual reasoning, or enjoy the challenge of abstract problems, you should consider enrolling in some **Pure Mathematics** units. They are wise choices not only for those whose principal interest lies in mathematics itself, but for all who wish to extend their reasoning ability: many students whose main interests lie in other disciplines find Pure Mathematics an ideal second major. A wide variety of pure units are offered, at both Advanced and Normal levels, covering all major branches of mathematics.

Pure mathematics

Broadly speaking, **pure mathematics** is mathematics motivated entirely for reasons other than application. It is distinguished by its rigour, abstraction and beauty. From the eighteenth century onwards, this was a recognized category of mathematical activity, sometimes characterised as *speculative mathematics*,[1] and at variance with the trend towards meeting the needs of navigation, astronomy, physics, engineering, and so on.

Subfields in pure mathematics

Analysis is concerned with the properties of functions. It deals with concepts such as continuity, limits, differentiation and integration, thus providing a rigorous foundation for the calculus of infinitesimals introduced by Newton and Leibniz in the 17th century. Real analysis studies functions of real numbers, while complex analysis extends the aforementioned concepts to functions of complex numbers. Functional analysis is a branch of analysis that infinite-dimensional vector spaces and views functions as points in these spaces.

Abstract algebra is not to be confused with the manipulation of formulae that is covered in secondary education. It studies sets together with binary operations defined on them. Sets and their binary operations may be classified according to their properties: for instance, if an operation is associative on a set which contains an identity element and inverses for each member of the set, the set and operation is considered to be a group. Other structures include rings, fields and vector spaces.

Geometry is the study of shapes and space, in particular, groups of transformations that act on spaces. For example, projective geometry is about the group of projective transformations that act on the real projective plane, whereas inversive geometry is concerned with the group of inversive transformations acting on the extended complex plane. Geometry has been extended to topology, which deals with objects known as topological spaces and continuous maps between them. Topology is concerned with the way in which a space is connected and ignores precise measurements of distance or angle.

Number theory is the theory of the positive integers. It is based on ideas such as divisibility and congruence. Its fundamental theorem states that each positive integer has a unique prime factorization. In some ways it is the most accessible discipline in pure mathematics for the general public: for instance the Goldbach conjecture is easily stated (but is yet to be proved or disproved). In other ways it is the least accessible discipline; for example, Wiles' proof that Fermat's equation has no nontrivial solutions requires understanding automorphic forms, which though intrinsic to nature have not found a place in Physics or in public discourse.

Applied mathematics

Applied mathematics is a branch of mathematics that concerns itself with the mathematical techniques typically used in the application of mathematical knowledge to other domains.

Divisions of applied mathematics

There is no consensus of what the various branches of applied mathematics are. Such categorizations are made difficult by the way mathematics and science change over time, and also by the way universities organize departments, courses, and degrees.

Historically, applied mathematics consisted principally of applied analysis, most notably differential equations, approximation theory (broadly construed, to include representations, asymptotic methods, variational methods, and numerical analysis), and applied probability. These areas of mathematics were intimately tied to the development of Newtonian Physics, and in fact the distinction between mathematicians and physicists was not sharply drawn before the mid-19th century. This history left a legacy as well; until the early 20th century subjects such as classical mechanics were often taught in applied mathematics departments at American universities rather than in physics departments, and fluid mechanics may still be taught in applied mathematics departments.

INVERSE GALOIS PROBLEM

In mathematics, the inverse Galois problem concerns whether or not every finite group appears as the Galois group of some Galois extension of the rational numbers Q. This problem, first posed in the 19th century, is unsolved.
More generally, let G be a given finite group, and let K be a field. Then the question is this: is there a Galois extension field L/K such that the Galois group of the extension is isomorphic to G? One says that G is realizable over K if such a field L exists.

Partial results

There is a great deal of detailed information in particular cases. It is known that every finite group is realizable over any function field in one variable over the complex numbers **C**, and more generally over function fields in one variable over any algebraically closed field of characteristic zero. Shafarevich showed that every finite solvable group is realizable over **Q**.

It also known that every sporadic group, except possibly the Mathieu group M_{23}, is realizable over **Q**.

Hilbert had shown that this question is related to a rationality question for G: if K is any extension of **Q**, on which G acts as an automorphism group and the invariant field K^G is rational over **Q**, then G is realizable over **Q**. Here rational means that it is a purely transcendental extension of **Q**, generated by an algebraically independent set. This criterion can for example be used to show that all the symmetric groups are realizable.

Much detailed work has been carried out on the question, which is in no sense solved in general. Some of this is based on constructing G geometrically as a Galois covering of the projective line: in algebraic terms, starting with an extension of the field **Q**(t) of rational functions in an indeterminate t. After that, one applies Hilbert's irreducibility theorem to specialise t, in such a way as to preserve the Galois group.

COMBINATORICS

Combinatorics involves the general study of discrete objects. Reasoning about such objects occurs throughout mathematics and science. For example, major biological problems involving decoding the genome and phylogenetic trees are largely combinatorial. Researchers in quantum gravity have developed deep combinatorial methods to evaluate integrals, and many problems in statistical mechanics are discretized into combinatorial problems. Three of the four 2006 Fields Medals were awarded for work closely related to combinatorics: Okounkov's work on random matrices and Kontsevich's conjecture, Tao's work on primes in arithmetic progression, and Werner's work on percolation.

Our department has been on the leading edge of combinatorics for the last forty years. The late Gian-Carlo Rota is regarded as the founding father of modern enumerative/algebraic combinatorics, transforming it from a bag of ad hoc tricks to a deep, unified subject with important connections to other areas of mathematics. Our department has been the nexus for developing connections between combinatorics, commutative algebra, algebraic geometry, and representation theory that have led to the solution of major long-standing problems. We've also been historically strong on the other parts of combinatorics, including extremal, probabilistic, and algorithmic combinatorics, some of which have close ties to other areas including computer science.

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